A Dynamic Near-Optimal Algorithm for Online Linear Programming

Yinyu Ye
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Joint work with Shipra Agrawal and Zizhuo Wang

Information-Based Complexity and Stochastic Computation

September 17, 2014
Outline

- Online Linear Programming
- Main Results and Key Ideas
- Related and More Recent Work
Consider a store that sells a number of goods/products
▶ There is a fixed selling period or number of buyers
Background

Consider a store that sells a number of goods/products
- There is a fixed selling period or number of buyers
- There is a fixed inventory of goods
Consider a store that sells a number of goods/products
▶ There is a fixed selling period or number of buyers
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▶ Customers come and require a bundle of goods and bid for certain prices
Consider a store that sells a number of goods/products

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- There is a fixed inventory of goods
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- Decision: To sell or not to sell to each individual customer?
Consider a store that sells a number of goods/products

- There is a fixed selling period or number of buyers
- There is a fixed inventory of goods
- Customers come and require a bundle of goods and bid for certain prices
- Decision: To sell or not to sell to each individual customer?
- Objective: Maximize the revenue.
# An Example

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<tr>
<th>Item</th>
<th>Bid 1 ((t = 1))</th>
<th>Bid 2 ((t = 2))</th>
<th>...</th>
<th>Inventory ((b))</th>
</tr>
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Online Linear Programming Model

The classical offline version of the above program can be formulated as a linear (integer) program as all information data would have arrived: compute $x_t$, $t = 1, \ldots, n$ and

$$\begin{align*}
\text{maximize}_x & \quad \sum_{t=1}^n \pi_t x_t \\
\text{subject to} & \quad \sum_{t=1}^n a_{it} x_t \leq b_i, \quad \forall i = 1, \ldots, m \\
& \quad x_t \in \{0, 1\} \ (0 \leq x_t \leq 1), \quad \forall t = 1, \ldots, n.
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Now we consider the online or streamline and data-driven version of this problem:

- We only know $b$ and $n$ at the start
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Now we consider the online or streamline and data-driven version of this problem:

- We only know $b$ and $n$ at the start
- The bidder information is revealed sequentially along with the corresponding objective coefficient.
- An irrevocable decision must be made as soon as an order arrives without observing or knowing the future data.
Application Overview

- Revenue management problems: Airline tickets booking, hotel booking;
- Online network routing on an edge-capacitated network;
- Online combinatorial auction;
- Online adwords allocation
Model Assumptions

Main Assumptions

- \( 0 \leq a_{it} \leq 1 \), for all \((i, t)\);
- \( \pi_t \geq 0 \) for all \( t \);
- The data \((a_t, \pi_t)\) arrive in a random order.

Denote the offline LP maximal value by \( \text{OPT}(A, \pi) \). We call an online algorithm \( A \) to be \( c \)-competitive if and only if
\[
E_{\sigma} \left[ \sum_{t=1}^{n} \pi_t x_{\left(\sigma, A\right)}(t) \right] \geq c \cdot \text{OPT}(A, \pi) \quad \forall (A, \pi),
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where \( \sigma \) is the permutation of arriving orders.
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where $\sigma$ is the permutation of arriving orders.

In what follows, we let

$$B = \min_i \{b_i\} (> 0).$$
Main Results: Necessary and Sufficient Conditions

Theorem

For any fixed $0 < \epsilon < 1$, there is no online algorithm for solving the linear program with competitive ratio $1 - \epsilon$ if

$$B < \frac{\log(m)}{\epsilon^2}.$$

Agrawal, Wang and Y (Operations Research 2014)

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Online LP, ICERM 2014
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The proof of the negative result is based on a distribution of instances (the number of each types of columns is chosen according to certain distribution) with $m = 2^k$, and then show that no allocation rule can achieve $(1 - \epsilon)$-optimality in expectation under randomized permutation.
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Key Ideas: A Learning Algorithm is Needed

The proof of the positive result is constructive and based on a learning policy.

- There is no distribution known so that any type of stochastic optimization models is not applicable.
- Unlike dynamic programming, the decision maker does not have full information/data so that a backward recursion cannot be carried out to find an optimal sequential decision policy.
- Thus, the online algorithm needs to be learning-based, in particular, learning-while-doing.
The problem would be easy if there is an "ideal price" vector:

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Pricing the bid: The optimal dual price vector $p^*$ of the offline LP problem can play such a role, that is $x_t^* = 1$ if $\pi_t > a_t^T p^*$ and $x_t^* = 0$ otherwise, yields a near-optimal solution.
Price Observation of Online Learning II

- **Pricing the bid**: The optimal dual price vector $\mathbf{p}^*$ of the offline LP problem can play such a role, that is $x_t^* = 1$ if $\pi_t > \mathbf{a}_t^T \mathbf{p}^*$ and $x_t^* = 0$ otherwise, yields a near-optimal solution.

- Based on this observation, our online algorithm works by learning a threshold price vector $\hat{\mathbf{p}}$ and using $\hat{\mathbf{p}}$ to price the bids.
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One-time learning algorithm: learn the price vector once using the initial $\epsilon n$ input.

Dynamic learning algorithm: dynamically update the price vector at a carefully chosen pace.
We illustrate a simple One-Time Learning Algorithm:

- Set $x_t = 0$ for all $1 \leq t \leq \epsilon n$;

Solve the $\epsilon$ portion of the problem

$$\max \sum_{t=1}^{\epsilon n} \pi_t x_t$$
subject to

$$\sum_{t=1}^{\epsilon n} a_{it} x_t \leq (1 - \epsilon) \epsilon b_i,$$
$$0 \leq x_t \leq 1,$$
$$t = 1, \ldots, \epsilon n$$
and get the optimal dual solution $\hat{p}$;

Determine the future allocation $x_t$ as:

$$x_t = \begin{cases} 
0 & \text{if } \pi_t \leq \hat{p}^T a_t \\
1 & \text{if } \pi_t > \hat{p}^T a_t 
\end{cases}$$
as long as $a_{it} x_t \leq b_i - \sum_{j=1}^{t-1} a_{ij} x_j$ for all $i$; otherwise, set $x_t = 0$. 

Yinyu Ye
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One-Time Learning Algorithm

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Yinyu Ye  Online LP, ICERM 2014
Theorem

For any fixed $\epsilon > 0$, the one-time learning algorithm is $(1 - \epsilon)$ competitive for solving the linear program when

$$B \geq \Omega \left( \frac{m \log (n/\epsilon)}{\epsilon^3} \right)$$
Outline of the Proof

- With high probability, we clear the market;
- With high probability, the revenue is near-optimal if we include the initial $\epsilon$ portion revenue;
- With high probability, the first $\epsilon$ portion revenue, a learning cost, doesn’t contribute too much.

Then, we prove that the one-time learning algorithm is $(1 - \epsilon)$ competitive under condition $B \geq \frac{6m \log (n/\epsilon)}{\epsilon^3}$.
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But this is one $\epsilon$ factor higher than the lower bound...
Dynamic Learning Algorithm

In the dynamic price learning algorithm, we update the price at time $\epsilon n$, $2\epsilon n$, $4\epsilon n$, ..., till $2^k \epsilon \geq 1$. 
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At time $\ell \in \{\epsilon n, 2\epsilon n, \ldots\}$, the price vector is the optimal dual solution to the following linear program:

$$\begin{align*}
\text{maximize}_{x} & \quad \sum_{t=1}^{\ell} \pi_t x_t \\
\text{subject to} & \quad \sum_{t=1}^{\ell} a_{it} x_t \leq (1 - h_{\ell}) \frac{\ell}{n} b_i \quad i = 1, \ldots, m \\
& \quad 0 \leq x_t \leq 1 \quad t = 1, \ldots, \ell
\end{align*}$$

where

$$h_{\ell} = \epsilon \sqrt{\frac{n}{\ell}};$$

and this price vector is used to determine the allocation for the next immediate period.
Geometric Pace/Grid of Price Updating

$t_1 = \epsilon n$
$t_2 = 2\epsilon n$
$t_3 = 4\epsilon n$
$t_4 = 8\epsilon n$
In the dynamic algorithm, we update the prices $\log_2 (1/\epsilon)$ times during the entire time horizon.
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The numbers $h_\ell$ play an important role in improving the condition on $B$ in the main theorem. It basically balances the probability that the inventory ever gets violated and the lost of revenue due to the factor $1 - h_\ell$. 
Comments on Dynamic Learning Algorithm

- In the dynamic algorithm, we update the prices $\log_2 \left( \frac{1}{\epsilon} \right)$ times during the entire time horizon.

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- Choosing large $h_\ell$ (more conservative) at the beginning periods and smaller $h_\ell$ (more aggressive) at the later periods, one can now control the loss of revenue by an $\epsilon$ order while the required size of $B$ can be weakened by an $\epsilon$ factor.
## Related Work on Random-Permutation

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Table: Comparison of several existing results
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Online Linear Programming

Main Results and Key Ideas

Related and More Recent Work

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- Buy-and-sell or double market?
Summary and Future Questions on OLP

- $B = \frac{\log m}{\epsilon^2}$ is now a necessary and sufficient condition (differing by a constant factor).
- Thus, they are near-optimal online algorithms for a very general class of online linear programs.
- The algorithms are distribution-free and/or non-parametric, thereby robust to distribution/data uncertainty.
- The dynamic learning has the feature of “learning-while-doing”, and is provably better than one-time learning by a factor.
- Buy-and-sell or double market?
- price-posting market?